ENACTIVISM AND MATHEMATICS EDUCATION

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The development of learning theories has continued since the acceptance of constructivism with enactivism being one emerging theory. Some influences on this development include criticisms of constructivism, ideas about Cartesian dichotomies, consideration of non-cognitive knowing, notions from phenomenology, and the neural biological work which emphasises evolutionary or Darwinian notions and systems theory. This paper, based on the literature, puts some of these interrelated influences together to introduce enactivism.

INTRODUCTION

In educational research there is a constant consideration of theories that provide possible frameworks for studies. This paper considers some of the criticisms of constructivism and some of the notions that underpin enactivism as part of this process. It is not a summary of enactivism but an attempt to provide an alternative way to make sense of what happens in classrooms and to tie together more coherently some of the research about learning, teaching and curriculum.

In the Anglo-American world, from 1930 to 1980, the dominant theories influencing mathematics education have been associationism and behaviourism. Minor influences have been the progressive education of Dewey (1916), and humanist education as espoused by writers like Neill (1968) and Rogers (1969). During this time, in other countries, theories such as Gestalt education (Wertheimer, 1980) have been influential.

Since the eighties we have seen a growth of interest in constructivist theories—radical and social constructivism (von Glasersfeld, 1995; Cobb, 1994) and variations of these. These have been valuable in questioning the assumptions made about what is knowledge, how people learn, and how we teach, assess, and present curriculum.

Starting from constructivism, enactivism seems to be emerging as a result of five interrelated influences which will be discussed in turn. These five are: the criticisms of constructivism, the Cartesian dichotomies, phenomenology, non-cognitive knowing, and issues from biology.

DISCUSSION OF CRITICISMS OF CONSTRUCTIVISM

While constructivism has been valuable, it has also been criticised. Five criticisms follow.

• There is a lack of models for constructivist teaching. This is hardly a criticism when the theory has always purported to be about learning rather than teaching, but related to this lack of a teaching model is a lack of curriculum and assessment models, and this has meant that teachers are subjected to mixed messages when they talk about constructivist learning but are presented with curricula and assessment in a behaviourist format.

• There is a lack of a critical dimension which means that there is no mechanism to avoid the construction of undesirable outcomes (Taylor, 1996). This seems no different than with traditional learning where the critical dimension depends mainly on the teacher.

• There is an undue influence in education and in what constitutes knowledge by the dominant culture which is currently the white middle class (Zevenbergen, 1996; Taylor, 1996). This concern about the hegemony of the dominant culture is relevant to theories other than constructivism could perhaps have been anticipated when the emphasis moved to social construction or enculturation. While the consideration of prior knowledge in constructivism and the emergence of ethnomathematics might suggest that this domination is being reduced, in fact there seems to be little evidence of change.

• Constructivism is concerned only with cognitive knowing. It does not explain unformulated or subconscious knowledge, it does not consider how things might be known intuitively or instinctively, and it does not consider how emotions are constructed or their role in learning.

• There do not seem to be explicit links made between constructivism and the learning theories that brain-science or neural biology offer.

ISSUES FROM PHILOSOPHY—CARTESIAN DICHOTOMIES

Related to these criticisms of constructivism are some philosophical assumptions that have been made about knowing that are now considered problematic. These include the dichotomy of self and non-self, or self and world, which lead to the subject/object, the mind/body, and the knower/known dichotomies (Davis, 1996). These have been referred to as Descartes' error (Damasio, 1994) but Davis (1996) says we should not denigrate Descartes' contributions, we need to see them in an historical context and as providing a way of seeing the world. He says that these Cartesian dichotomies have caused us to put more reliance on reason rather than on the senses and have led us to focus on building better and better representations of reality. These dichotomies of mind/body, self/other and self/world are not only Cartesian notions, their roots belong within the traditions of individualism in Aristotelian philosophy and Judaic and Christian religions. In these the dichotomies start with the body/spirit split which is reinforced by the notion of a vital principle or soul distinct from the body. This has connotations of intelligence not being connected with the material body.

In the phenomenology of Merleau-Ponty (1962) we see a reinterpretation of this Cartesian bipolar way of thinking that dominates western thinking. Merleau-Ponty rejects both the rationalist and the empiricist ways of knowing and claims that "the body renders mind and world inseparable" with the body being "our means of belonging to our world".

Varela, Thompson and Rosch (1991) suggest the same when they say that "cognition depends on the kinds of awareness that come from having a body with various sensorimotor capacities". Their argument contends that the individual and the environment must be considered together, that one can not separate knowledge from doing and from the body, and that knowing is doing which in the end is inseparable from self-identity or being (Varela, Thompson & Rosch, 1991). These notions of embodiment and systems theory (together with enactivism) seem to resonate more with Zen, Bhuddist and Taoist philosophies than with traditional western ones.

Tremmel (1993) is another educator who sees links between learning and Zen, he sees a need for learners to give themselves up to the learning situation and the action of the moment, he writes of the need to balance the external ecology with the internal ecology, and to study the self which is Zen.

Such an embodied view of knowing is not new, according to Boisvert (1998), Dewey had a similar problem with Cartesian philosophy and he also took what is now called a post-Darwinian or embodied view and did not accepted the mind/body split nor the subject/external world split.

PHENOMENOLOGY

Another aspect of philosophy that has influenced Maturana, Varela and Davis who are perhaps the most influential writers on enactivism is phenomenology. Phenomenology is a philosophy that was founded by Husserl and developed by Heidegger, Satre and Merleau-Ponty (Kvale, 1996). The basic notion in phenomenology is that phenomena are the objects of knowledge. It started with a focus on consciousness and awareness and was expanded to include the life-world and human action. It is concerned with both "that which appears

and the manner in which it appears", and it "attempts to get beyond immediately experienced meanings in order to articulate the pre-reflective level of lived meanings, to make the invisible visible" (Kvale, 1996). The question seems to be how are these phenomena internalised as knowledge.

Ideas from the biologist Maturana have been shown by Winograd and Flores (1987) to have many similarities to the ideas from the phenomenology of Husserl and Heidegger, although Maturana makes no reference to them. In particular Husserl believes that "the world is experienced (by me) only through the possibilities allowed by my ego" (Mingers, 1995) and Heidegger uses the phrase "being-in-the-world which summarises the notion that it is this process of experiencing the world that is the way of being of humans" (Minger, 1995), and these notions are similar to Maturana's ideas.

Varela, Thompson and Rosch (1991) have elaborated on the ideas of Merleau-Ponty about phenomenology and they claim that the body needs to be understood as both physical-biological, and at the same time, as experiential/phenomenological. They assume that the mind is influenced by both biological interactions and historical interactions.

NON-COGNITIVE KNOWING

Non-cognitive knowledge does not seem to be explained by constructivism. Davis (1996) suggests that this is because traditionally we have been concerned only with what he calls formulated knowledge because the cognising agent is seen as being separate from world. He discriminates between formulated and unformulated knowledge and says that much of what we do is unformulated as we are not conscious of doing it. He suggests that formulated and unformulated knowledge are not a dichotomy but are complementary and inseparable. He talks about what we say (the formulated) and what we do (the unformulated), and suggests that it is through the *play* between these that we find space for learning. Enactivism for Davis explains unformulated knowing because "every act is an act of cognition" and "we are not separate from but coupled to our situation/context". He summarises this with a phrase from Maturana and Varela (1987) "to live is to know".

Non-cognitive knowing is more than what Davis (1996) calls unformulated knowledge. Davis writes that learning involves resolving tensions between tacit and explicit knowledge, between emotional and reasoned actions, and between intuitive and calculated responses. He speaks of understanding implying sympathy, and meaning implying intent. He says that meaning has an affective dimension which is often ignored because of another Cartesian split, the knowing/feeling split. Other writers also emphasise some of these notions. Feminists writers have always considered emotions more than male writers have and Gilligan's (1982) notion of "connected knowledge" and the term "passionate knower" used by Belenkey, Clinchy, Goldberger and Tarule (1986) are examples of this. Hargreaves (1998) recently wrote that "emotions are at the heart of teaching". In saying this he quotes many other educators. However, for some of them emotions seem to be merely an extra dimension to be considered. Noddings (1992) puts emotions in a more central position when she talks of "the challenge to care in schools". Related to this are some comments from Heidegger, (cited in Mingers, 1995), in describing our experiencing of objects he talks of *dasein* which is characterised by a feeling of concern; and when describing our experience of other people, talks of *solicitude* which suggests a caring for and a concern for others. In a more general way Donaldson (1993) spoke of emotions when she wrote "passionate curiosity empowers the intellect", and "achievement of new understanding is normally accompanied by delight".

Whether we consider emotions as unformulated knowledge, as personal constructions, or as actions, there seems to be a need to consider them not apart from other constructions and actions but as part of our being which is intimately interrelated with our knowing.

ISSUES FROM BIOLOGY

The biological notions about learning have been influenced by the work:
- of cognitive scientists with models that compare human brains with computers;
- on linguistics, semiotics and language in thinking and learning (Pinker, 1994, 1997);
- in systems theory, and
- in neurobiology that is concerned about how the human brain works (Plotkin, 1998).

While each of these theories has distinctive features, many of them fit reasonably with the philosophical notions of constructivism. Within the biological studies concerned with how the mind works, the evolutionary metaphors of Darwin rather than the analytic and reductionist models of Descartes are gaining acceptance (Dennett, 1995).

Such evolutionary ideas in education are not new. According to Plotkin (1998), William James embraced the ideas of Darwinism in his early foundations of psychology but these notions were not taken aboard by his colleagues and virtually disappeared. People like Piaget with a background in biology were also sympathetic to notions of evolution, and the idea of *fit* from such educators is evidence of this.

Capra (1996) and Mingers (1995) have provided explanations of some of these ideas about learning that were developed by Varela and Maturana. Capra shows how the general systems theory from Bertlanffy (1968); the work of other biologists and ecologists, Gestalt psychology from the 30s, the studies by cyberneticians, and the development of self-organizing systems, underpin their work. Capra uses the metaphor of a *web* to explain the interrelatedness within and between living systems and says

> In the emerging theory of living systems mind is not a thing, but a process. It is cognition, the process of knowing, and it is identified with the process of life itself. This is the essence of the Santiago theory of cognition, proposed by Humberto Maturana and Francisco Varela (Capra, 1996, p 257).

Both Capra and Mingers describe Maturana's early work which focused on two question: "What is the organization of the living?" and "What takes place in phenomenon of perception?" and identify the breakthrough as seeing these two as intimately connected. He suggests that Maturana sees a living system as self-organizing and self-referring, and sees perception and cognition as not representing an external reality, but specifying one through the nervous system's organizational process.

According to Mingers (1995), Maturana explored these ideas in *The Biology of Cognition* (Maturana, 1970) and this led him to see humans as autopoietic (or self-producing) systems with nervous systems structurally coupled to bodies and through this to the environment. From this perspective "the body and the nervous system are structure-determined systems; the changes they undergo depend on their own prior structure and can only be triggered, not determined, by interactions with other systems" (Mingers, 1995). He goes on to describe how the capacity for language (including that of mathematics) has opened up new domains in which we make descriptions, descriptions of descriptions, descriptions of ourselves, leading to the emergence of the observer. Such descriptions are not determined by the nature of what is described, but by the describer, they do not reflect an objective reality, they remain subjective constructions. Summarizing Maturana (mainly from Maturana and Varela, 1980), Mingers makes the following points:

- All our perceptions and experiences occur through and are mediated by our bodies and nervous systems. It is therefore impossible for us to generate a description that is a pure description of reality, independent of ourselves.
- The structure- determined nature of the observer means that interactions are selective rather than instructive, so that the resulting experience always reflects the observer.
- We are therefore always, unavoidably, existing within a domain of subject-dependent

> descriptions—that is the only reality we can experience—the reality of the distinctions and descriptions that we make. We must accept this and alter our view of our world accordingly.
>
> - Thus the object of our knowledge does not exist except as it is distinguished by the observer. It is not just that we cannot access as existing reality, but that our realities are brought into existence through our activities as observers.
> - This is true not just of the everyday world of ordinary people, but also of the world of science itself, which is equally confined to exploring the results of its own distinctions.

According to Sacks (1995) who has provided a good overview of the work of Edelman (1987), Edelman sees the brain as likely to be very different from a computer and sees the need for a theory of how the brain works that accounts for or is compatible with "the facts of evolution, neural development and neurophysiology; and neurology and psychology". He sees this as needing to "be a theory of self-organisation and emergent order at every level and scale". Sacks claims that Edelman's theory does this. The theory is known as neural Darwinism or a theory of neuronal group selection where the nervous system (like the immune system) is a system of recognition and where evolutionary processes occur within each individual and within lifetimes. Edelman considers two kinds of selection: developmental (pre-birth or genetic instructions which provide a propensity rather than specific direction) and experiential (based on learning/life experiences).

Another scientist working in this area of neural biology is Plotkin. He (1994) has developed an evolutionary biologist's view of cognition, he sees knowledge as adaptation and believes that we think adaptively rather than logically. He suggests that we have a predisposition to pay attention to particular things and suggests that we might be hard wired for these. He disagrees with many biologists in that he sees language as being unique to humans and he sees it as a tool for learning. Plotkin also acknowledges the idea of emotional knowledge and cultural knowledge that suggest he is disagreeing with some of the cartesian notions. In his later work (1998) he pushes beyond the nature versus nurture argument by including the social context in his discussion of *nurture*. This means that nurture is influenced by historical occurrences in a similar way to the notion that nature (genes) are influenced by ancestors. Davis (1996) also does not see nurture and nature as distinct but sees the cognising agent as being an active rather than a passive agent who selects the actions to be taken.

ENACTIVISM

The criticisms of constructivism, the ideas from philosophy about Cartesian dichotomies and phenomenology, ideas of non-cognitive knowing, and the issues from biology have influenced the emerging theory about knowing, learning and teaching called enactivism. This theory is based on the ideas of Maturana and Varela (1987) and has been developed by Davis (1996) who has interpreted it within the context of mathematics education.

In enactivism, instead of seeing learning as "coming to know", one envisages learner and learned, knower and known, self and other, as co-evolving and being co-implicated. In this situation context is neither the setting for a learning activity, nor the place where the student is, but the student is literally part of the context. With enactivism the complexity of learning is emphasised:

> learning should not be understood in terms of a sequence of actions, but in terms of an ongoing structural dance—a complex choreography—of events which, even in retrospect, cannot be fully disentangled and understood, let alone reproduced (Davis, Sumara and Kieren, 1996, p 153)

Enactivism contrasts with constructivism where there is an emphasis on knowing rather than on knowledge. With constructivism knowledge is viewed as a human construction that is evaluated in terms of whether it fits with the experience of the knower. The representation people construct of their realities can be viewed as a process of redefinition

which occurs when ideas are transformed in a social setting so that they can be stabilized, inspected, edited, and shared by others. In this situation viability is the criteria and this fits with the neo-Darwinian idea of *natural drift* rather than *natural selection* (Varela, Thompson and Rosch, 1991). Even with the radical constructivism of von Glasersfeld (1995), while the emphasis is on individual interpretation of and abstractions from experience, these are acknowledged as being shaped by the learning context, by interaction with others, and by the social milieu. According to Davis (1996), radical constructivism emphasises the monologic authority of the individual and social constructivism emphasises the dialogic authority of the society and the individual, but both these are based on the modernist separation of self from other and self from world. Both, Davis claims, have difficulty because they see knowledge as something, and therefore need to assign it a location. An alternative to this need to locate knowledge is provided by Bateson (1972) who says that there is no such thing as information, it is not knowledge-as-object but knowledge-as-action.

This emphasis on action ties in with the way Davis (1996) sees it, he says that in enactivism collective action is not for individual sense-making but as a location for shared meanings and understanding. In radical constructivism understanding is seen as a continuous process of organizing one's knowledge structures, but Davis (1996) describes cognition as being not in minds and brains, but in the possibility for shared action. He talks of cognition/knowledge and knowers as being co-implicated and co-emerging. Knowledge is not apart from world but embedded in it in a series of increasingly complex systems (groups, schools, communities, cultures, humanity, biosphere), and embodied knowledge extends to these bodies that are larger than human. Varela (1992) reinforces this, he sees enactivism as providing an alternative to the constructivists' notion of representation by focussing on self-organizing systems. From this viewpoint he questions the existence of a world independent of the knower and sees the knower, the knowing and the known as emerging together. Clark (1998) describes emergence as complementing reductionist analysis as a way of making sense of this 'embodied' cognition. From his perspective an explanation can be said to emerge in a similar way to the way that the result emerges from a dynamic system (in chaos theory)—from a small start, gathering momentum and direction as more inputs 'fit' with the initial starting point.

Within enactivism Davis (1996) talks about 'self'. He says that self is a 'transformative' process. Piaget saw self as continually changing but enactivists see change as not happening to self but as being self. Davis sees self as a network of relationships and says that one does not don different masks, but one enacts different selves.

In summary, while the rationalist says 'I think', the empiricist says 'I observe', and the enactivist says 'I act'. Whether we accept the constructivist knowledge schema, the web notion of Capra, or the enactivist theory of Davis as our way of thinking about coming to know, we see how we 'come to know' as being more complex than was implied by a simple list of objectives. A similar perspective of complexity is needed when we look at curriculum and assessment.

CONCLUSION

While this paper was written to provide some background to enactivism, being concerned with practice, many educators will say "so what". Davis (1996) has taken the next step and in his book he teases out some of the main requirements for teaching that he sees as being congruent with enactivism.

His most important requirement is listening—not at the shallow level but at a deep level. He discriminates between evaluative listening (the traditional evaluative role taken by

teachers), interpretive listening (which cuts through the noise of 'play' and leads to more flexibility in the classroom), and hermeneutic listening (which involves more negotiation and co-implicated activity within the classroom).

This listening is often situated in a 'play' situation which on the one hand reminds one of the emphasis on play in the work of Dewey and of Piaget But Davis also asserts that play is a situation where 'subjectivity loses itself' and play is, in Gadamer's words, 'not the opposite of seriousness' but rather "seriousness in playing is necessary to make play wholly play' (Gadamer, 1990). Davis's (1996) notion of play recognizes that play only exists in the playing, he sees that when playing around we use 'body time' and get totally involved. He sees play as an essential human quality that is evident in all we do and is something that can be realized in stillness, and in solitude.

In terms of curriculum Davis (1996) writes about curriculum anticipating. This means that the teacher works from good learning activities but must anticipate different ways that the lesson might move in response to the students' interactions while still linking with the major ideas that underpin the particular curriculum.

It would be presumptuous for me to try to summarise Davis's book, or to assume that what he calls enactivism is the only meaning that can be ascribed to it. However I would recommend his book as a critique of and a source book for teaching based in the context of mathematics; it provides some ways 'towards a sound alternative'; and is a reference that develops the notions of enactivism further.

Just as I started with some of the critiques of constructivism, so this explanation of enactivism will no doubt be critiqued. This paper was only intended as an introduction to the theory and it is hoped that readers will continue to suspend their judgement until they have delved deeper into the literature and considered other aspects of the theory.

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